MATH 111*i* WORKSHEET 4

Name: Solutions

Work in groups to answer as many problems as you can. Ask questions if you get stuck.

1. In each of the following equations, solve for the variable.

(a) $13 + 2(1 - u) = 8u - 5(u + 7)$

$\Rightarrow 13 + 2 - 2u = 8u - 5u - 35$

$\Rightarrow 15 + 35 = 3u + 2u$

$\Rightarrow 50 = 5u$

Answer: $u = 10$

(b) $8(2 + 3z) + 1 = z - 10(z + 1)$

$\Rightarrow 16 + 24z + 1 = z - 10z - 10$

$\Rightarrow 17 + 10 = -9z - 24z$

$\Rightarrow 27 = -33z$

Answer: $z = -9/11$

(c) $8 - (4 - 12t) + 2 = 3t + 2(7 - 3t)$

$\Rightarrow 10 - 4 + 12t = 3t + 14 - 6t$

$6 - 14 = -3t - 12t$

$-8 = -15t$

Answer: $t = 8/15$

(d) $2x(6x - 1) + 21 = 8x - x(3 - 12x)$

$\Rightarrow 12x^2 - 2x + 21 = 8x - 3x + 12x^2$

$\Rightarrow -2x + 21 = 5x$

$\Rightarrow 21 = 7x$

Answer: $x = 3$

(e) $\frac{6x+24}{x+4} = 5$ → $x \neq -4$

$6x+24 = 5(x+4)$

$6x - 5x = 20 - 24$

$x = -4$

But $x \neq -4$

Answer: No solution

(f) $\frac{6t-1}{t^2+5t+4} = -\frac{19}{t+1}$ → $t \neq -1, -4$

$\Rightarrow \frac{6t-1}{(t+1)(t+4)} = -\frac{19}{t+1}$

$\Rightarrow \frac{6t-1}{(t+1)(t+4)} = \frac{-19(t+4)}{(t+1)(t+4)}$

$\Rightarrow 6t-1 = -19t - 76$

$\Rightarrow 25t = -75$

Answer: $t = -3$

2. Solve the following equations for the indicated variable.

(a) Solve $A = 3p(4-2r)$ for p.

$\Rightarrow \frac{A}{4-2r} = 3p$

$\Rightarrow p = \frac{A}{3(4-2r)}$

Answer: _____

(c) Solve $A = 3p(4-2r)$ for r.

$\Rightarrow \frac{A}{3p} = 4-2r$

$\Rightarrow \frac{A}{3p} - 4 = -2r$

$\Rightarrow 2 - \frac{A}{6p} = r$

Answer: _____

(b) Solve $T = \frac{c}{3}\left(6p + \frac{3q}{c}\right) - 7p$ for p.

$\Rightarrow T = 2pc + q - 7p$

$\Rightarrow T - q = (2c-7)p$

Answer: $p = \frac{T-q}{2c-7}$

(d) Solve $T = \frac{c}{3}\left(6p + \frac{3q}{c}\right) - 7p$ for c.

$\Rightarrow T = 2pc + q - 7p$

$\Rightarrow T + 7p - q = 2pc$

Answer: $c = \frac{T+7p-q}{2p}$

Cont.

(e) Solve $3A + 6C = 4A(B - 7C)$ for C.

$\Rightarrow 3A+6C = 4AB - 28AC$

$\Rightarrow 6C + 28AC = 4AB - 3A$

$\Rightarrow (6+28A)C = (4B-3)A$

$\Rightarrow C = \dfrac{(4B-3)A}{2(3+14A)}$

Answer:

(h) Solve $3A + 6C = 4A(B - 7C)$ for A.

$\Rightarrow 6C = 4A(B-7C) - 3A$

$= (4(B-7C)-3)A$

$\Rightarrow A = \dfrac{6C}{(4(B-7C)-3)}$

Answer:

(f) Solve $y = \dfrac{4-9x}{3}$ for x.

$\Rightarrow 3y = 4 - 9x$

$\Rightarrow 9x = 4 - 3y$

Answer: $x = \dfrac{4-3y}{9}$

(i) Solve $y = \dfrac{12}{1-x}$ for x.

$\Rightarrow (1-x) = \dfrac{12}{y}$

$\Rightarrow x = 1 - \dfrac{12}{y}$

Answer: $x = \dfrac{y-12}{y}$

(g) Solve $y = \dfrac{7}{10x+9}$ for x

$\Rightarrow 10x+9 = 7/y$

$\Rightarrow 10x = 7/y - 9$

$= \dfrac{7-9y}{y}$

Answer: $x = \dfrac{7-9y}{70}$

(j) Solve $y = \dfrac{8-5x}{9-7x}$ for x.

$\Rightarrow (9-7x)y = (8-5x)$

$\Rightarrow 9y - 7xy = 8 - 5x$

$\Rightarrow 5x - 7xy = 8 - 9y$

$\Rightarrow x(5-7y) = 8 - 9y$

Answer: $x = \dfrac{8-9y}{5-7y}$

Cont.

3. Determine the number of distinct roots for each of the following polynomials. Do not find the roots.

(a) $25x^2 - 120x + 619 = 0$

$b^2-4ac = 120^2 - 4(25)(619) < 0$

Answer: 0

(b) $104x^2 - 75x - 14 = 0$

$b^2-4ac = 75^2 - 4(104)(-14)$
$= 75^2 + 4(104)(14) > 0$

Answer: 2

(c) $2x^2 + 60x + 450 = 0$

$b^2-4ac = 60^2 - 4(2)(450)$
$= 60^2 - 4(2)(30)(15)$
$= 60^2 - (60)(60) = 0$

Answer: 1

(d) $7x^2 - 17x + 5 = 0$

$b^2-4ac = 17^2 - 4(7)(5)$
$= 289 - 140 > 0$

Answer: 2

(e) $\frac{1}{6}x^2 - 43 = 0$

$b^2-4ac = 0 - 4(\frac{1}{6})(-43$
$= \frac{4}{6}(43) > 0$

Answer: 2.

(f) $97 + 136x + 289x^2 = 0$

$b^2-4ac = 136^2 - 4(289)(97) < 0$

Answer: 0

(g) $10x^2 - 7x = 0$

$b^2-4ac = 0 - 4(10)(-7)$
$= 4(10)(7) > 0$

Answer: 2

(h) $\frac{49}{9}x^2 + \frac{14}{15}x + \frac{1}{25} = 0$

$b^2-4ac = \frac{196}{225} - \frac{4(49)(1)}{225}$
$= 0$

Answer: 1

4. Solve each of the following equations.

(a) $x^2 - 4x + 3 = 0$

$\Rightarrow (x-1)(x-3) = 0$

Answer: $x = 1, 3$

(b) $4x^2 + x - 3 = 0$

$\Rightarrow (4x-3)(x+1) = 0$

Answer: $x = 3/4, -1$

Cont.

(c) $x+6=(x+4)^2$

$\Rightarrow x^2+8x+16=x+6$

$\Rightarrow x^2+7x+10=0$

$\Rightarrow (x+2)(x+5)=0$

Answer: $x=-2,-5$

(d) $(x+3)^2=11(x+3)$

$\Rightarrow x^2+6x+9=11x+33$

$\Rightarrow x^2-5x-24=0$

$\Rightarrow (x-8)(x+3)=0$

Answer: $x=8,-3$

(e) $x-8=\sqrt{22-3x}$

$\Rightarrow (x-8)^2=22-3x$

$\Rightarrow x^2-16x+64=22-3x$

$\Rightarrow x^2-13x+42=0$

$b^2-4ac=13^2-4(42)<0$

Answer: No solution

(f) $2-x=\sqrt{8-7x}$

$\Rightarrow (2-x)^2=8-7x$

$\Rightarrow x^2-4x+4=8-7x$

$\Rightarrow x^2+3x-4=0$

$\Rightarrow (x+4)(x-1)=0$

Answer: $x=1,-4$

(g) $\sqrt{1+3x}=4+\sqrt{5-x}$

$x\neq 1$

$\Rightarrow \sqrt{1+3x}-\sqrt{5-x}=4$

$\Rightarrow 1+3x+5-x-2\sqrt{(1+3x)(5-x)}=16$

$\Rightarrow -2\sqrt{(1+3x)(5-x)}=2(5-x)$

$\Rightarrow (1+3x)(5-x)=(5-x)^2$

$\Rightarrow (5-x)[(5-x)-(1+3x)]=0$

$\Rightarrow (5-x)(4-4x)=0 \Rightarrow x=1,5$

$\Rightarrow (5-x)(1-x)=0$

Answer: $x=5$

(h) $\sqrt{x-3}+\sqrt{x+1}=2$

$\Rightarrow x-3+x+1+2\sqrt{(x-3)(x+1)}=4$

$\Rightarrow 2\sqrt{(x-3)(x+1)}=2(3-x)$

$\Rightarrow (x-3)(x+1)=(3-x)^2$

$\Rightarrow x=3$

Answer:

Cont.

5. Solve each of the following inequalities. Give two solutions; one in inequality form and one in interval notation form. Ex. $a < x \leq b$ and $(a, b]$.

(a) $7x + 2(4 - x) < 12 - 3(5 + 6x)$

$\Rightarrow 7x + 8 - 2x < 12 - 15 - 18x$

$\Rightarrow 23x < -11$

Answer: $x < \frac{-11}{23}$

Answer: $(-\infty, -11/23)$

(b) $10(3 + w) \geq 9(2 - 4w)$

$\Rightarrow 30 + 10w \geq 18 - 36w$

$\Rightarrow 46w \geq -12$

Answer: $w \geq -6/23$

Answer: $[-6/23, \infty)$

(c) $2(4 + 5y) \leq 12y - 6(1 - 3y)$

$\Rightarrow 8 + 10y \leq 12y - 6 + 18y$

$\Rightarrow 14 \leq 20y$

Answer: $y \geq 7/10$

Answer: $[7/10, \infty)$

(d) $\left(\frac{1}{3} - \frac{1}{6}z\right) > \frac{1}{9}z + 4\left(2 - \frac{7}{18}z\right)$ $\times 18$

$\Rightarrow 6 - 3z > 2z + 4(36 - 7z)$

$\Rightarrow 6 > 5z + 144 - 28z$

$\Rightarrow -138 > -23z$

Answer: $z > 6$

Answer: $(6, \infty)$

(e) $-4 < 7x + 8 \leq 1$

$\Rightarrow -12 < 7x \leq -7$

Answer: $-\frac{12}{7} < x \leq -1$

Answer: $\left(-\frac{12}{7}, -1\right]$

(f) $2 \leq 2 + 4(3 - x) \leq 6$

$\Rightarrow 0 \leq 4(3 - x) \leq 4$

$\Rightarrow 0 \leq 3 - x \leq 1$

$\Rightarrow -3 \leq -x \leq -2$

Answer: $2 \leq x \leq 3$

Answer: $[2, 3]$

Cont.

(g) $-4 < 7x + 8 \leq 1$

Answer: $[-1/11, 16/11]$

(h) $\frac{1}{2} < 2\left(\frac{1}{4} + \frac{1}{8}t\right) < \frac{3}{4}$

Answer: $0 < t < 1$

Answer: $(0, 1)$

(i) $-12 \leq 4 - 11m \leq 3$

Answer: $\frac{1}{11} \leq m \leq \frac{16}{11}$

(j) $0 \leq \frac{3}{7} - \frac{5}{14}x < \frac{1}{2}$

Same as e)

oops

Answer: $-1/5 < x \leq 6/5$

Answer: $(-1/5, 6/5]$

(k) $-8 < 2(3 + 4x) - 4(1 + 3x) \leq 3$

Answer: $-1/4 \leq x < 5/2$

Answer: $[-1/4, 5/2)$

6. Solve the following inequalities.

(a) $z^2 - 11z + 24 < 0$

Answer: $3 < z < 8$

Cont.

(b) $2x^2 - 3 \geq 5x$

Answer: $(-\infty, -\frac{1}{2}] \cup [3, \infty)$

(c) $t^2 > 30 - 7t$

Answer: $(-\infty, -10) \cup (3, \infty)$

(d) $m^2 - 7m \leq 8$

Answer: $[-1, 8]$

(e) $x^2 + 6x \geq -9$

Answer: All real numbers

(f) $u^2 + u \leq 1$

Answer: $\left[\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right]$

(g) $w^2 + 4w - 12 > 0$

Answer: $(-\infty, -6) \cup (2, \infty)$

Cont.

(h) $x^2 + 49 > 14x$

Answer: $x \neq 0$

(i) $t^2 \leq t$

Answer: $[0, 1]$

(j) $x^2 - 8x > -14$

Answer: $(-\infty, 4-\sqrt{2}) \cup (4+\sqrt{2}, \infty)$

(k) $9u^2 - 6u + 1 < 0$

Answer: No solutions

(l) $z^6 + 8z^5 + 12z^4 \geq 0$

Answer: $(-\infty, -6] \cup [-2, \infty)$

(m) $2w^3 - 3w^2 < 14w$

Answer: $(-\infty, -2) \cup (0, 7/2)$

The End.